Anisotropic Creep Modeling for f.c.c. Single Crystals

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Summary
The one-dimensional behavior of single crystal superalloys at high temperatures under constant and cyclic creep conditions is described by means of a 4-parameter rheological model based on linear viscoelasticity. Tertiary creep is taken into account by reducing the effective cross section by means of an additional damage parameter. Tensile creep tests have been used for the identification of the material constants by a non-linear optimization procedure. For the generalization to three dimensions, a complete tensor-representation of cubic material symmetry is given. It contains twelve (temperature dependent) material parameters. Some results by finite element analysis will be presented.

Introduction
Within the last decade, single crystal superalloys have become more and more important for blading within the hot section of gas turbines. Because of their f.c.c. crystal structure the material behavior is highly orientation dependent and, hence, must be treated as (cubic) anisotropic. At high temperatures the inelastic properties under constant and cyclic loads are of paramount interest for the design of blades. Creep experiments show us that, after a short phase of primary creep, the secondary or steady creep phase almost immediately turns over into the overlinear phase of tertiary creep. Because of the absence of grain boundaries in single crystals, the creep deformation can reach about 20% so that nonlinear geometrical effects are to be considered at that stage, although the physical nonlinearities due to creep damage are predominant.

One-dimensional theory
As starting point for our analysis we choose a four-parameter model from linear viscoelasticity which consists of two elasticities and two viscosities (Fig.1). The governing differential equation in the stress $\sigma$ and the tension $\varepsilon$ which completely describes the behavior of the model is

$$\frac{d^2\sigma}{dx^2} + a_1 \frac{d\sigma}{dx} + a_2 \sigma = a_3 \frac{d\varepsilon}{dt} + a_4 \varepsilon,$$

where the four (positive) material constants are given by

$$a_1 = C / D + C / L + K / L,$$
$$a_2 = (C K) / (L D),$$
$$a_3 = C + K,$$
$$a_4 = C K / D.$$

International Union of Theoretical and Applied Mechanics

M. Życzkowski (Ed.)

Creep in Structures
4th IUTAM Symposium, Cracow, Poland
September 10-14, 1990

Springer-Verlag
Berlin Heidelberg NewYork London Paris Tokyo
Hong Kong Barcelona Budapest
Reduced form separation is limited only that the dominant term in (7) is the first one, so that a

As a first step, these data were used to determine the constants in (9) and (10) by a

Deep loads:

Large scatter of experimental results at the same stress level, which is typical for

In equation (10) we assume the evolution function of the damage parameter

where $\delta_0$ is the apparent cross section area, $\Delta$ is the effective (or critical) area

Damage modeling

In the two evolution relations (7) are applicable for material integration and may

We introduce a range creep behavior. Let final failure we introduce

![Graphical representation of damage modeling](image)

By introducing the stress in the lower branch of the model as an internal variable

![Diagram of the rheological model](image)
gives similar results. The curves in Fig. 4 show these calculated creep curves which prove the capability of the model to describe creep behavior in the full range of these tests.

![Graph showing creep curves for different stress levels.](image)

Fig. 4. Creep tests at 760 C, Experiment and calculation.

In order to do a global identification, i.e. to give a single set of material constants that describe all of the creep tests in the given range, we suggest the following procedure. First, select one test as a master curve for the others (in our case we chose KVS1) and identify C, K, D, L, and c0 by fitting this single test. Second, introduce a linear time transformation by

$$ t^* = t / \alpha $$

where \( t \) is the real time, \( t^* \) is an artificial or internal time, and \( \alpha \) is a time scaling factor, that depends on the initial stress \( \sigma_0 \). In our case the linear form

$$ \alpha = (k_1 \sigma_0 + k_2) $$

(10)

gives satisfying results. We now solve the evolution equations (4) with respect to the internal time \( t^* \) and afterwards transform the solution to the real time by means of (9) and (10). The results are given in Fig. 5 together with a selected set of creep tests. The parameter list is given in Table 1.

<table>
<thead>
<tr>
<th>C</th>
<th>K</th>
<th>D</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPa</td>
<td>GPa</td>
<td>GPa</td>
<td>h</td>
</tr>
<tr>
<td>50.52</td>
<td>49.64</td>
<td>57.69</td>
<td>6775.</td>
</tr>
</tbody>
</table>

![Graph showing creep with and without damage.](image)

Fig. 5. Global adjustment of creep tests KVS1, 40, 44, 50.

![Graph showing influence of nonlinearity.](image)

Fig. 6. Influence of nonlinearity.
It is easy to make (10) explicit in terms of stress increments if one prefaces the octahedral stress jump with a decoupling relationship of the irreducible form:

\[ \sigma_i = \sigma_i^e - \sigma_i^r \]

where \( \sigma_i^e \) and \( \sigma_i^r \) are the elastic and plastic components, respectively.

\[ \sigma_i^e = C : \varepsilon_i^e \]

with

\[ C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \]

and

\[ \varepsilon_i^e = \frac{1}{2} \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \]

where \( \nabla \mathbf{u} \) is the gradient of the displacement field. The stress jump can be expressed as

\[ \sigma_i^r = \frac{1}{2} \left( \mathbf{T} : \mathbf{A} \right) \]

where \( \mathbf{T} \) is the second stress invariant and \( \mathbf{A} \) is the fourth-order tensor.

\[ \mathbf{T} = \frac{1}{2} \left( \sigma_i + \sigma_j \right) \]

and

\[ \mathbf{A} = \frac{1}{2} \left( \sigma_i \sigma_j - \frac{1}{3} \sigma_k \sigma_k \mathbf{I} \right) \]

where \( \sigma_i \) and \( \sigma_j \) are the principal stresses.

The appropriate choice of damage constants (as being done in Figs. 4 and 5)...

The effective stress jump can be obtained using the effective stress approach. This allows for a simple relationship between the effective stress and the damage parameter. The difference between the two approaches is negligible for large deformation ratios.

\[ \Delta \sigma = \frac{1}{2} \left( \sigma_1 - \sigma_2 + \sigma_3 \right) \]

where \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \) are the principal stresses.
obtained by using ADINA [6] for tensile cyclic creep test of different orientations. The twelve material constants were only roughly fitted to experimental results. In BERTRAM et al. [7] applications to notched specimens are shown. A three-dimensional version of the damage equation will not be given here.

References


Acknowledgement

This report was supported by the Bundesminister für Forschung und Technologie under grant O3 M 3005 C4.