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CONSTITUTIVE MODELLING OF PRIMARY AND SECONDARY CREEP OF SINGLE CRYSTALS AT HIGH TEMPERATURES

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Introduction

Among the great amount of creep theories, few attempts can be found that describe the three dimensional behavior of anisotropic solids like nickel-base superalloy single crystals with f.c.c. structure. The suggestions in this field can be roughly divided into two groups. First, 1-d constitutive creep laws of the octahedral and cubic crystallographic slip systems are composed to a three dimensional (uncoupled) model (11/11/11). And, second, the cubic representations of tensor functions and ordered expansions are used to develop three dimensional laws (8/8/8/8). Our approach lies between the two methods. We assume a constitutive law from linear viscoelasticity theory by means of a rheological model and generalize it in order to describe the three-dimensional behavior by a complete cubic representation. The result is a twelve parameter model. A material identification for special orientations was done by an optimization procedure using cyclic tension tests. Thus, we are able to describe the primary and secondary creep behavior and to demonstrate some effects due to the anisotropy.

One dimensional theory

The following rheological model will be used to describe the one dimensional creep behavior of superalloys at high temperatures:

\[
\sigma = \sigma' + \left( \frac{C}{D} + \frac{K}{L} \right) \varepsilon' + \frac{K}{DL} \varepsilon = (C + K) \varepsilon' + \frac{K}{D} \varepsilon' \tag{1}
\]

which can be integrated in order to obtain an explicit version for the rate of deformations

\[
\varepsilon(\alpha) = \frac{1}{L} + \frac{1}{D} \left( \frac{C}{C+K} \right)^2 \frac{\partial \sigma}{\partial t} + \frac{1}{C+K} \frac{\partial \sigma}{\partial t} - \frac{K}{C+K} \int_0^{\alpha} \frac{\partial \sigma}{\partial t} \frac{C K / D}{C+K} (\alpha - \alpha) \, d\alpha \tag{2}
\]

if the initial state is stress-free. We obtain a more practical form, which is appropriate for computational purposes, if we introduce the force in the lower branch of the model as an internal variable.
\[
\dot{\sigma} = K \int_0^t \left( \varepsilon(t') - \frac{\sigma(t')}{L_0} \right) \, dt
\]

It satisfies the evolution equation

\[
s^* = \frac{K}{C+K} \left( a^* + \frac{C}{D} (a-o) \right)
\]

whereas the incremental form of the material law turns out to be

\[
e^* = \frac{1}{C+K} \left( a^* + \frac{C}{D} \left( \frac{C}{L} + \frac{K}{L} \right) (a-o) - \frac{C}{D} s \right)
\]

The numerical integration of these equations is depicted in Fig.1 together with the experimental results (dashed line). The material constants are determined by the latter by means of a nonlinear optimization technique.

![Graph showing experimental and calculated deformation in cyclic creep test.](image)

**Fig.1**

One-dimensional plot of experimental (dashed line) and calculated deformation in cyclic creep test.

**Three-dimensional constitutive laws**

In order to generalize the one-dimensional constitutive equations, we have to satisfy the symmetry properties of the cubic-anisotropic single crystal. The pure elastic (Hooke’s) law of cubic symmetry can always be brought into the form

\[
T = c_1 \varepsilon^\text{sym} + c_2 E_2 + c_3 E_3
\]

with the CAUCHY stress tensor \( T \), the identity tensor \( I \), and the linear deformation tensor \( E := \frac{1}{2} (\text{Grad} u \cdot \text{Grad} u) \), which is decomposed into

\[
E = E_1 + E_2 + E_3
\]

according to

\[
E_1 := \frac{1}{2} \varepsilon^\text{sym} I ; E_2 := \sum e_{ii} e_i \otimes e_i - E_1 ; E_3 := E - E_1 - E_2
\]
(summation over the three crystal directions). This decomposition is orthogonal:

$$\tau_i(n_i, n_j) = 0 \text{ for } i \neq j.$$  \hspace{1cm} (9)

If we introduce the linear projection tensors of fourth order

$$P_1 = \frac{1}{4} I \otimes I \quad P_2 = I - \sum e_i \otimes e_i \otimes e_i \otimes e_i \quad P_3 = \sum e_i \otimes e_i \otimes e_i \otimes e_i \cdot \frac{1}{4} I \otimes I$$  \hspace{1cm} (10)

we obtain the comprehensive relation

$$E_i = P_i[K] \quad i=1,2,3$$  \hspace{1cm} (11)

and we can define the fourth order elasticity tensor

$$C_i = 3c_1 P_1 + c_2 P_2 + c_3 P_3$$  \hspace{1cm} (12)

such that

$$T = C[E]$$  \hspace{1cm} (13)

holds. This representation is the most general of a linear mapping which identically fulfills the cubic symmetry properties and, hence, will be used in what follows in order to generalize the viscoelastic differential equation (2):

$$T'' + C_1[T'] + C_2[T] = C_3[K'] + C_4[E']$$  \hspace{1cm} (14)

so that each of the fourth order tensors $C_i$ has the same form as the elasticity tensor (12) before

$$C_i = 3a_{11} P_1 + a_{12} P_2 + a_{13} P_3$$  \hspace{1cm} (15)

with

$$a_{11} = C_1/D_1 + C_2/l_1 + K_1/l_1 \quad a_{12} = C_1 K_1/D_1 l_1$$  \hspace{1cm} (16)

$$a_{13} = C_1 + K_1 \quad a_{14} = C_1 K_1/D_1$$

By introducing the tensor of the internal variables like (3)

$$S = \Sigma_i K_i P_i \int_0^t \{E(t)'' \cdot Id_{l_1} + T(t)\} \, dt$$  \hspace{1cm} (17)

we obtain the incremental forms in analogy to equs. (4), (5)

$$K' = \left( \Sigma_i K_i (l_1 + K_1) P_i \right) [T'] + \left( \Sigma_i a_{11} (l_1 + K_1) P_i \right) [T] - \left( \Sigma_i C_i/D_1 l_1 + K_1 \right) [S]$$

$$S' = \left( \Sigma_i K_i (l_1 + K_1) P_i \right) [T'] + \left( \Sigma_i (C_i/D_1 l_1 + K_1) P_i \right) [T - S]$$  \hspace{1cm} (18)

which are more convenient for numerical purposes. They contain the 12 material constants $C_i, K_i, l_i, D_i, \quad i=1,\ldots,3.$ If they are all non-negative, then by means of the basic rheological model, compatibility with the second law of thermodynamics is assured. They can be determined by experimental results of samples with different orientations.

The 3-d version of the proposed constitutive law (18) has been implemented in ADINA 10. The material constants used in the finite element analysis are given in the table.
In Fig. 2 we plotted the results of the computer-simulation of creep-tests under constant loads. Fig. 3 gives the first interval of a cyclic creep test. The plots of a cylindrical bar of [0 0 1] and [0 1 2] orientation under tension performs the well-known phenomena due to anisotropy such as tension-torsion coupling, and evaluation of the cross section (Fig. 4).
Fig. 4
Tension of cylindrical bars (above), their cross sections (below) in [0 0 1] (left) and [0 1 2] (right) orientations.
Conclusions

The suggested constitutive equations describe the general three-dimensional cubic behavior of the model. Its remaining 12 non-negative constants are to be determined by experimental results in different crystallographic directions. The suggestion of composing a linear viscous behavior on all the octahedral and cubic slip systems is included here as a special case.

References


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